Derivujem podľa x

$$\frac{\partial \psi_k(x,t)}{\partial x} = ik\,\psi_k(x,t)$$

$$\frac{\partial^2 \psi(x,t)}{\partial x^2} = (-ik)^2\psi_k(x,t) = -k^2\psi_k(x,t) \Big/ \cdot \left(-\frac{\hbar^2}{2m}\right)$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x,t)}{\partial t^2} = \frac{\hbar^2k^2}{2m}\psi_k(x,t) = \underline{E_k\psi_k(x,t)} \qquad (2)$$

(1) a (2) majú rovnaké pravé strany

⇒ musia mať rovnaké aj ľavé

$$\boxed{i\hbar\frac{\partial\psi_k(x,t)}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi_k(x,t)}{\partial x^2}} \qquad (3)$$

Hrubou silou sa možno presvedčiť, že toto platí aj pre superpozíciu stavov

$$\psi(x,t) = \sum_k c_k\psi_k(x,t) = \sum_k c_k e^{i(kx-\omega_k t)}$$

V troch rozmeroch

$$\psi_{k_1k_2k_3}(\vec{x},t) = e^{i(k_1x_1+k_2x_2+k_3x_3-\omega_k t)}$$

kde $E_k = \hbar\omega_k = \frac{\hbar^2}{2m}\left(k_1^2+k_2^2+k_3^2\right)$

⇒ platí

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\psi \qquad (4)$$

Opäť priamo možno overiť platnosť pre superpozíciu.

Zavedieme:

$$\Delta = \frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$$